



MAS 117

Introduction to Algebra

Assignment 9

For handing in on 21 March 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

1 (30 marks) Let R be a ring in which $a^2 = a$ for all $a \in R$. By considering $(a + b)^2$, show

(a) R is commutative;

(b) $a + a = 0$ for all $a \in R$.

(Such a ring is called a *Boolean ring*. *Hint*: Show first that $ab = -ba$ for all $a, b \in R$.)

2 (40 marks) Which of the following systems $(G, \circ)$ are groups? Justify your answer.

(a) G is the set of all subsets of a set S , $\circ$ is intersection.

(b) G is the set of all rotations about the origin of the real Euclidean plane, $\circ$ is composition of mappings.

(c) $G = 2\mathbb{Z}$ (the set of even integers), $\circ$ is addition.

(d) $G = 2\mathbb{Z}$, $\circ$ is multiplication.

(e) $G = \mathbb{Z}$, $\circ$ is subtraction.

3 (30 marks) Let G be a group with four elements $\{e, a, b, c\}$, in which e is the identity, and $a^2 = b$. Write down the Cayley table of G , explaining your working.

4 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

A group G contains five elements u, v, w, x, y (none of which is the identity element) satisfying

$$u \circ v = w, \quad v \circ w = x, \quad w \circ x = y, \quad x \circ y = u, \quad y \circ u = v.$$

What is the order of u ?